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Original article

Quality adjusted price indexes and the Willig condition

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Abstract

This paper provides a graphical explanation of Willig's condition for developing consistent price indexes for quality change. The framework also describes how the single crossing condition generalizes the Willig condition.

Keywords: Quality adjustment; Price indexes; Weak complementarity; Willig condition

JEL classification: D11; H41

Over twenty-five years ago, Willig (1978) demonstrated that observable demand data allowed consistent adjustments to price indexes for quality changes. The Marshallian consumer surplus for a small quality change, per unit of the private good experiencing that quality change, can serve as a price index adjustment for improvements or reductions in quality. To use his measure in practice, preferences must be restricted to control the effects of income changes on these Marshallian measures. This restriction assures they can serve as price equivalents. Meeting this need requires the conditions he derived for ignoring the income effects. Virtually all of the recent discussions of quality adjusted price indexes–including the special sections of both the Journal of Economic Perspectives (1998, 2003) and the Journal of Business and Economic Statistics (1999), papers by

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Nevo (2003) and Pakes, Berry, and Levinson (1993), as well as Bresnahan and Gordon's (1997) edited NBER volume on quality and new goods, and even the background documents for the Final Report of the *Advisory Committee to Study the Consumer Price Index*–ignore his results (see US Senate [1996]).[2]

One reason why his paper may have been overlooked stems from the fact that his central result, Theorem 1, has never been connected to a simple analysis of how quality adjusted price changes would be constructed. This paper explains his Theorem and illustrates how price indexes are linked to quality with a simple figure.

Willig's Theorem 1 relates to a nonessential good X which is a weak complement to its quality.[3] His result demonstrates that three statements are equivalent when an incremental Marshallian surplus for a small quality change is convergent and differentiable with respect to income (m). The three conditions are:

(a) $\dfrac{1}{X(p,q,m)}\displaystyle\int_p^\infty X_q(t,q,m)\mathrm{d}t$ is independent of income;

(b) $\dfrac{V_q}{V_p}$ is independent of income; and

(c) $-\dfrac{V_q}{V_p}=\dfrac{1}{X(p,q,m)}\displaystyle\int_p^\infty X_q(t,q,m)\mathrm{d}t.$

In this description, $X(.)$ is the Marshallian demand for the weak complement with $X_q=\frac{\partial X}{\partial q}$. Condition (a) implies the derivative of Marshallian consumer surplus with respect to quality, averaged over the quantity of the weak complement demanded at the benchmark price, quality level, and income, is independent of income. Palmquist (2005) has re-interpreted the Willig condition, identifying it with path independence of the line integral of the demand and virtual price functions in p–q space, which in turn implies the following equality: $V_q(p)/V_m(p)=\int_p^\infty X_q(t,q,m)\mathrm{d}t$. The term on the left side of this equation is the Marshallian virtual price of q. $V_q(p)$ is intended to represent the partial derivative of the indirect utility, $V(p,q,m)$, with respect

[2] Trajtenberg (1989) and Banzhaf (2002) are the only papers we could locate among the set dealing with quality adjusted prices that identify Willig (1978) in the references. Trajtenberg's paper does not describe how Willig's results relate to his welfare measures for product innovations. Instead, it assumes the income effects are zero. Banzhaf (2002) simulates actual price adjustments in a way that is consistent with Willig's results. Mäler (1974) and Bradford and Hildebrandt (1977) are credited with introducing the concept.

[3] Weak complementarity is defined for situations where one good (or quality attribute) makes no contribution to an individual's utility unless a positive amount of its associated good is consumed. Two examples illustrate the concept involved: (a) improvements in the horsepower of an automobile are not valued without consumption of the automobile; and (b) increases in the quality of a lake's water are not valued unless the individual uses the lake.

[4] Bockstael and McConnell (1993) describe this equality as implying the quality change can be equivalently evaluated with the Marshallian virtual price or the double integral of change in the Marshallian demand function with respect to price and quality.

$$\int_{q_0}^{q_1}\frac{V_q(p,s,m)}{V_m(p,s,m)}\mathrm{d}s=\int_{q_0}^{q_1}\int_p^\infty X_q(t,s,m)\mathrm{d}t\mathrm{d}s$$

Palmquist's derivation highlights how the Willig condition can be considered as comparable to the symmetry condition for prices in defining consumer surplus for multiple price changes.

Marshallian consumer surplus for a quality change defined using the demand function for the weak complement (the right side of the above equation) remains a well-defined concept. We could also define it using the Marshallian virtual price and the two measures need not be the same. It is the Willig condition (here defined following Palmquist's argument as a symmetry condition) that assures measurement using the integral of the Marshallian virtual price function will yield the same measure as the area between two Marshallian demands for the weak complement distinguished by the quality change involved.

to q, evaluated at the price, p, corresponding to the lower limit of the integral.[4] Condition (b) implies the slope of an indirect utility function's indifference curve in q–p space is independent of income. Its relationship to a price index follows when we note that the slope is $-(V_q/V_p)=\mathrm{d}p/\mathrm{d}q$. Since the Willig condition implies (V_q/V_p) is independent of income, $\mathrm{d}p/\mathrm{d}q$ is as well. Thus, the compensating price adjustment for a change in q will be independent of income when the Willig condition is satisfied. Finally, (c) specifies that the Marshallian consumer surplus for a small change in quality per unit of the weak complement is equivalent to $\mathrm{d}p/\mathrm{d}q$. This result implies when the Willig condition is satisfied a measurable concept, the Marhshallian consumer surplus for a quality change per unit of the weak complement, provides the basis for estimating the price adjustment to compensate for quality changes.

This description does not separately identify the prices of other goods because they are assumed constant for describing the theorem. Willig's Theorem 4 observes that under conditions (a) and (b) of his Theorem I, price adjustments for quality changes do not need to be based on compensated income (his Lemma 1). "Hedonic" adjustments to prices can be defined from the information provided by ordinary demand functions. As we noted, they can be based on the Marshallian consumer surplus for quality per unit of X consumed, provided this ratio is independent of income.

A number of authors have offered analytical interpretations for these results. Bockstael and McConnell (1993), for example, focus on the symmetry condition implied by Theorem 1. That is, $-\frac{\partial X}{\partial q}=\frac{\partial \pi^{\mathrm{M}}}{\partial p}$, where π^{M} is the Marshallian virtual price for quality (V_q/V_m) and $\partial X/\partial q$ is the quality slope of the Marshallian demand for the weak complement.[5] As noted earlier, Palmquist (2005) demonstrates that this symmetry condition parallels the path independence condition used to define consistent consumer surplus measures for multiple price changes. Neither paper offers an intuitive explanation of how the three components of Theorem 1 are important to using the Willig condition for quality adjusted price indexes.

To meet this objective we graph weak complementarity and the Willig condition's effects on the tradeoffs between the weak complement (X) and a numeraire good (Z). Fig. 1 combines two aspects of our argument. The lower indifference map, originating on the Z axis at R (i.e. $u(q_{\mathrm{o}})$ and $u(q_1)$), illustrates how the X versus Z tradeoff is affected by q when X is a weak complement for q. That is, when $X=0$, q has no value, so the indifference curves drawn to represent the same utility (i.e. $u(q_{\mathrm{o}})=u(q_1)$) but different levels of q (with $q_1>q_0$) will all intersect at the vertical axis at R. In a two dimensional diagram, the indifference curves appear to "fan out" from this point (Smith and Banzhaf, 2004). The price change represented by the pivot in the budget constraint from TS to TS' is the *Hicksian equivalent price* change for the quality change from q_0 to q_1. That is, the quality improvement from q_0 to q_1 requires an increase in the relative price of X from TS to TS' to assure utility is unchanged. The average $1/2(AB+CD)$ is a first order approximation of the Hicksian consumer surplus for either the quality change or the equivalent price change. The Marshallian consumer surplus is found by constructing an indifference curve for q_1, $u*(q_1)$, that corresponds to a different level of utility but holds quality constant at q_1 and can be realized with prices and income level defined by TS. $1/2(CD+EF)$ provides a first order approximation of the Marshallian surplus for the same Hicksian price change or equivalently the utility preserving quality change.[6] Thus, Willig's measure of the quality adjustment to the price of the weak complement, described using the graph, is $1/2(CD+EF)/X$. Geometrically, this

[5] As Palmquist (2005) notes, the minus sign is omitted from their development.

[6] The first order approximation for consumer surplus for a movement along the Marshallian demand equivalent to this price change is $(p_1-p_0)X^*+1/2(p_1-p_0)(X-X^*)=1/2(p_1-p_0)(X+X^*)$. The result follows from the fact that CD is $(m-p_0X^*)-(m-p_1X^*)$ and EF is $(m-p_0X)-(m-p_1X)$ and from substituting.

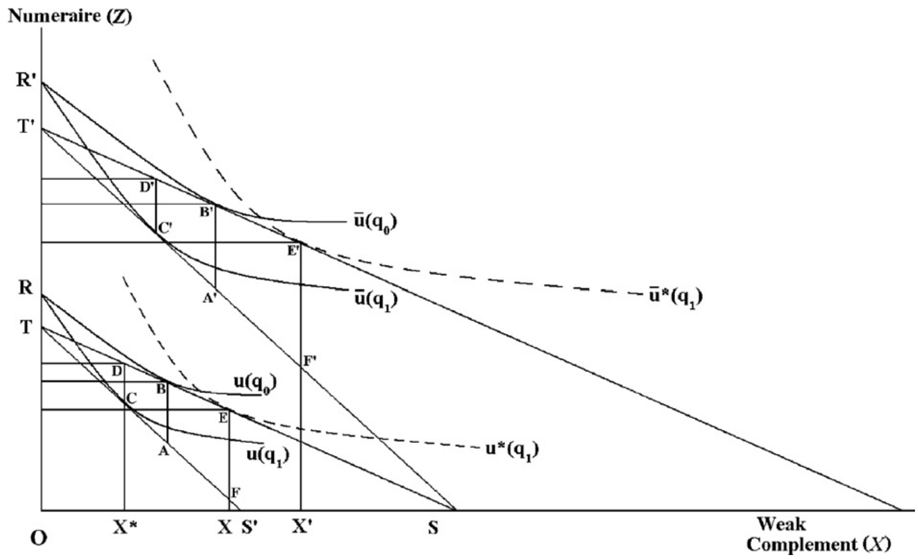

Fig. 1. Quality change, income change, and the Willig condition.

measure is the change in slope of the budget line price from TS to TS' (or expressed as a positive value, $\frac{OT}{OS'}-\frac{OT}{OS}$). Since TDC and TEF are similar triangles, by a triangle proportionality theorem, this change in price is $1/2(CD+EF)/1/2(X^*+X)$. That is, the Marshallian consumer surplus divided by the average consumption over the range. (The difference in the denominator from Willig's notation arises only for discrete changes diagrammed; his result follows from the differential calculus around point X, e.g. $X^*=X-\mathrm{d}X$.)

Finally, Willig's theorem requires that $1/2(CD+EF)/X$ be constant with income changes. Fig. 1 illustrates this point by reproducing the indifference map for the same quality change at a higher level of income (point T' versus T for the initial income). The increase in X due to the income change from T to T' needs to be proportional to the increase in MCS, or in terms of Fig. 1,[7]

$$\frac{(X'-X)}{X}=\frac{(E'F'+C'D')-(EF+CD)}{(EF+CD)}.$$

This proportionality assures that $\mathrm{d}p/\mathrm{d}q$ will be invariant with respect to income.[8]

This condition has either been incorrectly ignored (Hausman, 2003) or assumed away (Trajtenberg, 1989). In the later case, Willig's result implies the author could have derived a quality adjusted price index with less restrictive assumptions. Integrating the demand function over own price will not capture the full effect of quality changes on Hicksian surplus measures of a quality change (Bockstael and McConnell, 1993). Recently, Bullock and Minot (in press) have corrected a misconception in the literature. Prior to their numerical analysis, the Willig (1978) condition was viewed as the only means to adapt the Hausman logic

[7] This relationship also explains Palmquist's (2005) interpretation of Willig as requiring the income elasticity of demand for the weak complement (X) to equal the price flexibility of income for quality (q).

[8] This result can be described in intuitive terms for the linear case as follows: $\frac{MCS}{X}=1/2(p(q_1)-p(q_0))\cdot\left(\frac{X+X'}{X}\right)$. As (q_1-q_0) approaches zero, $1/2\left(\frac{X+X'}{X}\right)$ approaches unity and $p(q_1)-p(q_0)$ approaches $\mathrm{d}p/\mathrm{d}q$.

to derive Hicksian measures for quality changes.[9] They demonstrate that an adaptation to the Vartia (1983) results can provide a numerical measure of the Hicksian adjustment to income to derive the willingness to pay for a quality change. Their logic requires that the three conditions used in their numerical algorithm satisfy a generalized form of the implicit function theorem.[10]

Recent work has demonstrated it is possible to replace Willig's restriction with structural models. These models fully parameterize preferences, making it possible to identify Hicksian as well as Marshallian constructs of all types, and to identify the income adjustments that Willig's restriction helps to avoid. However, they do have the disadvantage of imposing more structure on the problem.

Willig's Theorem 1 is a required condition for analytically deriving the quality adjusted price indexes from market demand curves. Surprisingly, it has been almost completely ignored in the price index literature, where it should play an important role. As we noted earlier, it relaxes the requirement for zero income effects (Trajtenberg, 1989). On the other hand, it would add to the conditions required to use Hausman's (1999) method to adjust price indexes from shifts in the demand of the weak complement.[11] We have provided a simple diagram to motivate the importance of the condition and to explain it intuitively. If we can assume weak complementarity and the Willig condition are satisfied, then the price index for quality is defined by the Marshallian consumer surplus per unit of the good experiencing the change and will be invariant with income.

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[9] We stated this conclusion in Smith and Banzhaf (2004) as an implication of Bockstael and McConnell (1993).

[10] See von Hafen (2006) for further discussion of limitations in their approach.

[11] It is also one way to use Hausman's (1981) differential equations to recover quasi expenditure functions capable of evaluating quality changes. In principle, Bullock and Minot (in press) approach could be used to numerically derive a Hicksian price index for quality.

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